Stochastic acceleration in blazars

Andrea Tramacere
Outline

• Phenomenological signatures
• setup of Theory/Numerical framework for stochastic acceleration
• Self-consistent reproduction of Long Term Trends
• numerical modeling, numerical fit (no eyeball fit) no analytical approximations
SPECTRAL DISTRIBUTION OF HBLs

\[ S(E) = S_p \times 10^{-b \left( \log(E/E_p) \right)^2} \]

- **b**: curvature at peak
- **E_p**: peak energy
- **S_p**: SED height @ **E_p**
acceleration signature in the $E_s$-vs-$b$ trend

PKS 0548-322, 1H1426+418, Mrk 501, 1ES1959+650, PKS2155-34

Ep-vs-b, different scenarios

Long term (overall 13 years of data)
Ep-vs-b trends hint for an acceleration dominated scenario
acceleration signature in the $E_S$-vs-$L_S$ trend

long-trend main drivers

$\gamma_3 p \uparrow$ and $n(\gamma_3 p) \downarrow$ => $\alpha < 1.5$

acceleration + energy conservation

$B \rightarrow \alpha = 2.0$, incompatible as long-trend main driver

$\delta \rightarrow \alpha = 4$ long-trend main driver

Tramacere+2011
Mrk 501 1997 Flare

Massaro & Tramacere +2006

7 Apr 1997

16 Apr 1997

Hard spectra $s << 2.00$

\[ s = 1 + \frac{t_{acc}}{2t_{esc}} \]

**best fit pars**

**best-fit parameters:**

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LP+PL spectra
Synch index~[1.6-1.7]=⇒s~[2.2-2.4]

Fermi I+Fermi II  Mrk 421 2006

Log(νF(ν))
erg cm⁻² s⁻¹

Log(v) (Hz)

Tramacere +2009

Lemoine,Pelletier 2003

γ_min
r
γ_c

ε/ε₀

ε/ε₀
Mrk 421 2009 data

data from Abdo et al 2011
Fermi-LAT+Magic coll.

\[
\begin{align*}
\text{lppl/plc} & \quad \text{p-value}= 6.8\text{E-6}
\end{align*}
\]
The log-parabola origin: physical insight
The origin of the log-parabolic shape: statistical derivation

\[ \varepsilon = \bar{\varepsilon} + \chi \]

\( \varepsilon_i \) is a R.V.

\[ \gamma_{n_s} = \gamma_0 \Pi_{i=1}^{n_s} \varepsilon_i \]

C.L. Theorem multipl. case

systematic

log-normal distribution

Log-Parabolic representation

\[ \log(n(\gamma)) \propto \frac{(\log \gamma - \mu)^2}{2\sigma^2_{\gamma}} \propto r \left[ \log(\gamma) - \mu \right]^2 \]
\[
\frac{\partial n(\gamma, t)}{\partial t} = \frac{\partial}{\partial \gamma} \left\{ - [S(\gamma, t) + D_A(\gamma, t)] n(\gamma, t) + D_p(\gamma, t) \frac{\partial n(\gamma, t)}{\partial \gamma} \right\} - \frac{n(\gamma, t)}{T_{esc}(\gamma)} + Q(\gamma, t)
\]

analytical solution for:
\[D_p \sim \gamma^q, \; q=2\]

“hard-sphere” case no cooling

Melrose 1968,

\[
n(\gamma, t) = \frac{N_0}{\gamma \sqrt{4\pi D_{p0} t}} \exp \left\{ - \frac{[\ln(\gamma/\gamma_0) - (A_{p0} - D_{p0})t]^2}{4D_{p0}t} \right\}
\]
set-up of the accelerator

\[ t_A \sim \gamma^2 \]

\[ \lambda_{\text{max}} \]
\[ K_{\text{min}} \]
\[ \gamma_{\text{max}} \]
\[ \lambda_{\text{coh}} \]
\[ K_{\text{coh}} \]
\[ \gamma_{\text{coh}} \]

\[ r_g, \lambda, \gamma \]

hard-sphere \( q=2 \)

non resonant regime
resonant regime
energy-dependent

\[ W(k) \]
spectral trends

single flare
Pile-up and hard spectra

\( q = 2, R = 10^{15} \text{ cm}, B = 0.1 \text{ G}, t_{\text{inj}} = t_D = 10^4 \text{ s} \)

Mrk 501 1997

\[ s \approx 1.6 \]

\( r \approx 0.7 - 0.8 \ll r_{\text{eq}} \approx 6 \)

\( s \ll s_{\text{FI}} - 2.3 \)
Pile-up and hard spectra

Mrk 501 2014 Flare MAGIC paper (submitted)
<table>
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<td>spectral shape</td>
<td>LPPL or LP</td>
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spectral trends

multiple flares and population trends
**$E_s$-$b_s$ X-ray trend and $\gamma$-ray predictions**

- **Dp trend**
  - Data span 13 years, both flaring and quiescent states
  - We are able to reproduce these long-term behaviours, by changing the value of only one parameter ($D_p$)
  - For $q=2$, curvature values imply distribution far from the equilibrium ($b\sim[1.0-0.7]$)
  - More data needed at GeV/TeV, curvature seems to be cooling-dominated
  - Similar trend observed in GRBs (Massaro & Grindlay 2001)

**γ-ray TeV**
- Mrk 421 - MAGIC 2006 and 2008

**Parameters' Values Adopted in the Numerical Solutions of the Diffusion Equation**

- $L_{inj}$ ($E_s$-$b_s$ trend) (erg s$^{-1}$): 5 × 10$^{39}$
- $L_{inj}$ ($E_s$-$L_s$ trend) (erg s$^{-1}$): 5 × 10$^{38}$, 5 × 10$^{39}$
- $q$: 2
- $t_A$ (s): 1.2 × 10$^3$
- $t_{D0} = 1/D_{P0}$ (s): [1.5 × 10$^4$, 1.5 × 10$^5$]
- $T_{inj}$ (s): 10$^4$
- $T_{esc}$ ($R/c$): 2.0
**ES-LS X-ray trend and γ-ray predictions**

- The ES–SS (ES–LS) relation follows naturally from that between ES and bs.
- The low L_{inj} objects (Mrk 501 vs Mrk 421) reach a larger ES, compatibly with larger γ_{eq}.
- Mrk 421 MAGIC data on 2006 match very well the Synchrotron prediction with simultaneous X-ray data.
- The average index of the trend L_{S} ∝ E_{S}^{α} with α ∼ 0.6, is compatible with the data, and with a scenario in which a typical constant energy (L_{inj} x t_{inj}) is injected for any flare (jet–feeding problem), whilst the peak dynamic is ruled by the turbulence in the magnetic field.
JetSeT Documentation

JetSeT is an open source C/Python framework to reproduce radiative and accelerative processes acting in relativistic jets, allowing to fit the numerical models to observed data. The main features of this framework are:

- handling observed data: re-binning, definition of data sets, bindings to astropy tables and quantities definition of complex numerical radiative scenarios: Synchrotron Self-Compton (SSC), external Compton (EC) and EC against the CMB

- Constraining of the model in the pre-fitting stage, based on accurate and already published phenomenological trends. In particular, starting from phenomenological parameters, such as spectral indices, peak fluxes and frequencies, and spectral curvatures, that the code evaluates automatically, the pre-fitting algorithm is able to provide a good starting model, following the phenomenological trends that I have implemented. fitting of multiwavelength SEDs using both frequentist approach (iminuit) and bayesian MCMC sampling (emcee)

- Self-consistent temporal evolution of the plasma under the effect of radiative and accelerative processes, both first order and second order (stochastic acceleration) processes.

Jet SeT modeler and fitting Tool

Author: Andrea Tramacere
The Astrophysical Journal

numerical models fit

MCMC sampler

Temp. ev. of the plasma

minuit plugin
backup slides
injection term

\[ L_{inj} = \frac{4}{3} \pi R^3 \int \gamma_{inj} m_e c^2 Q(\gamma_{inj}, t) d\gamma_{inj} \ (\text{erg/s}) \]

systematic term

\[ S(\gamma, t) = -C(\gamma, t) + A(\gamma, t) \]

cooling term

\[ C(\gamma) = |\dot{\gamma}_{\text{synch}}| + |\dot{\gamma}_{\text{IC}}| \]

syst. acc. term

\[ A(\gamma) = A_{p0} \gamma, \ t_A = \frac{1}{A_0} \]

\[
\frac{\partial n(\gamma, t)}{\partial t} = \frac{\partial}{\partial \gamma} \left\{ - \left[ S(\gamma, t) + D_A(\gamma, t) \right] n(\gamma, t) + D_p(\gamma, t) \frac{\partial n(\gamma, t)}{\partial \gamma} \right\} - \frac{n(\gamma, t)}{T_{\text{esc}}(\gamma)} + Q(\gamma, t)
\]

Turbulent magnetic field \[\rightarrow\] momentum diffusion term

\[ W(k) = \frac{\delta B(k_0^2)}{8\pi} \left( \frac{k}{k_0} \right)^{-q} \]
Mrk 501 2014 Flare
MAGIC paper (submitted)

cont. single injection (Stawarz&Petrosian 2009)
not compatible with MW data

double cospatial injection
compatible with data

model parameters:

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S vs IC

![Graph showing the relationship between S vs IC](image)

- **S averaged spectra**
- **IC averaged spectra**
- **SED peak**
- **PL fit. index = 0.6**
- **PL fit. index = 0.6**
- **PL fit. index = 1.0**

**Figure 10.**

**Tramacere +2011**

![Graphs showing E_s (keV) and E_c (GeV)](image)

- **TH regime**
- **KN regime**

**Equation 32: a=0.68**

**PL fit. index = -0.14**

![Graph showing curvature b_s vs. v peak S (Hz)](image)

![Graph showing curvature b_c vs. v peak IC (Hz)](image)
blazars in a nutshell

2. blazar population samples
2.1. Swift/BAT
The Swift satellite is a N(S multix wavelength observatory. The Burst (lert Telescope on board Swift is a coded mask instrument operating in the 3–8 keV energy range. The coded mask is made of 43 lead tiles arranged in a random half-open/half-closed pattern. (T performs continuous surveys of the hard X-ray sky, accumulating sky maps every 4 minutes. For our study we selected all the objects that were flagged as class 4 in the 47 months Swift catalog. In table 1 the sample used is reported, together with their 3–8 keV flux, their photon indices and their redshifts.

2.2. INTEGRAL/I BIS/ISGRI
The INTErnational GammaRay (strophysics Laboratory is the latest hard X-ray/soft x-ray mission of the ES(w fruit of a collaboration with IKI and N(S. The Imager on board the INTEGRAL satellite is designed to obtain high resolution images.
IC cooling and equilibrium

\[ R = 1 \times 10^{15} \text{ cm} \]

\[ R = 5 \times 10^{13} \text{ cm} \]

\[ U_{\text{ph}} (R = 1 \times 10^{13} \text{ cm}) \gg U_{\text{ph}} (R = 1 \times 10^{15} \text{ cm}) \]

IC prevents higher energies in more compact accelerators (if all the parameters are the same) **Impact on rapid TeV variability!**
S vs IC

Tramacere +2011

- S averaged spectra
- IC averaged spectra
- SED peak
- PL fit, index = 0.6
- PL fit, index = 0.6
- PL fit, index = 1.0

Tramacere +2011

- Fit by Eq. 32, a = 0.68
- PL fit, index = -0.14

TH regime

KN regime

- V peak S (Hz)
- E_c (GeV)
- Curvature b_s
- Curvature b_c
The effect of the turbulence index \( q \) on the diffusive shock acceleration time \( t_D \), which is given by

\[
t_D = \frac{1}{D_{p0}} \left( \frac{\gamma}{\gamma_0} \right)^{2-q}
\]

where \( D_{p0} \) is the momentum-diffusion coefficient, \( \gamma \) is the Lorentz factor, and \( \gamma_0 \) is a reference Lorentz factor. The plot illustrates the variation of \( t_D \) with the logarithm of \( \gamma \) for different values of \( q \): for \( q < 2 \) (Kolmogorov/Kraichnan turbulence), \( t_D \) decreases with increasing \( \gamma \), and for \( q = 2 \) (hard-sphere approximation), \( t_D \) is independent of \( \gamma \).
effect of the turbulence index $q$

$B=1.0 \, G, \, t_{D0}=10^3, \, R=5\times10^{15} \, cm$

- $q=2$
  - hard-sphere
  - $r_{eq} \approx 6$
  - $b_{eq} \approx 0.7$

- $q=5/3$
  - Kolmogorov
  - $r_{eq} \approx 6$

- $q=3/2$
  - Kraichnan
  - $r_{eq} \approx 6$

$n(\gamma)$ curvature

synch. peak curvature

Kolmogorov

Kraichnan

hard-sphere
At first, for simplicity, we consider the effect of each process viewed separately on the energy spectrum, and then the simultaneous effect of two or more processes.

Spectra of Isolated Processes

1. Random and Systematic Acceleration.

The kinetic equation is

\[
\frac{\partial N}{\partial t} = \alpha_1(t) \frac{\partial}{\partial E} \left( E^2 \frac{\partial N}{\partial E} \right) - \alpha_2(t) \frac{\partial}{\partial E} (EN). 
\]

Let the energy distribution be specified, at each instant of time \( t_0 \), by the \( \delta \)-function in the neighborhood of energy \( E_0 \):

\[
N(E, 0) = N_0 \delta(E - E_0) \quad \text{and} \quad \int_{E_0}^{E_0} N(E, 0) \, dE = N_0.
\]

Then, utilizing the techniques developed, e.g., in [13], we may find that

\[
N(E, t) = \frac{N_0}{\sqrt{\pi E_2 \sqrt{a_1}}} e^{- \left( \ln \frac{E}{E_0} + a_1 + a_2 \right)^2 / 4a_1}, \quad (1)
\]

where

\[
a_1 = \int_{t_0}^{t} \alpha_1(t) \, dt, \quad a_2 = \int_{t_0}^{t} \alpha_2(t) \, dt.
\]

At \( E_{\text{max}} \)}
where $n(\gamma) = \frac{N_0}{\gamma \sigma_\gamma \sqrt{2\pi}} \exp \left[ -\left( \frac{\ln(\gamma/\gamma_0) - n_s \left[ \ln \bar{\epsilon} - \frac{1}{2} (\sigma_\epsilon / \bar{\epsilon})^2 \right]}{2n_s(\sigma_\epsilon / \bar{\epsilon})^2} \right]^2 \right]$. 

The curvature $r$ is inversely proportional to $t \rightarrow n_s$ and $D_p \rightarrow \sigma_\epsilon$. 

log-parabolic shape natural consequence of dispersion
b distributions and $q$

both flaring and quiescent seem to be far from equilibrium $b \approx [0.7-1.0]$ (if full KN or S)

compatible with $q=2$ far from equilibrium $b_{eq} \approx 0.7$

$q=2$ require more fine tuning, especially on duration

$q=2$ far from equilibrium $b_{eq} \approx 0.7$

$q=3/2$

$q=5/3$

$q=2$ require more fine tuning, especially on duration
self-consistent approach: \textbf{acc+cooling}

\[ t_D = \frac{1}{D_{p0}} \left( \frac{\gamma}{\gamma_0} \right)^{2-q} \]

\[ t_{DA} = \frac{1}{2D_{p0}} \left( \frac{\gamma}{\gamma_0} \right)^{2-q} \]

\textbf{observed values}

- \[ E_{p1}/E_{p2} \sim 5 \]
- \[ \Delta t \sim \text{few ks} \]

values compatible with Tammi & Duffy 2009

- \[ t_{DA} \sim< 5 \text{ ks} \]
- \[ t_D \sim< 10 \text{ ks} \]

\textbf{set-up of the accelerator}

- \( R \sim 10^{13}-10^{15} \text{ cm} \)
- \( \delta B/B << 1, B \sim [0.01-1.0] \text{ G} \)
- \( \beta_A \sim 0.1-0.5 \)
- \( \lambda_{\text{max}} < R \Rightarrow \sim 10^{9-15} \text{ cm} \)
- \( \rho_g < \lambda_{\text{max}} \Rightarrow \gamma_{\text{max}} \sim 10^{7.5} \)

\[ R \]

\[ \lambda_{\text{max}} \]

\[ \rho_g \]
Flare: acc.-dominated-vs-equil., $R = 10^{15}$ cm, $q=2$

- mono energetic inj., $t_{\text{inj}} << t_{\text{acc}}$, $t_{\text{inj}} << t_{\text{sim}}$
- we measure $r@\text{peak}$ as a function of the time
- two phase: acceleration-dominated, equilibrium
- equil. distribution:
  - $f=1$ for $q=2$ and S, full TH, or full KN
  - equil. curv.: $r \sim 2.5$, ($r_{3p} \sim 6.0$) for TH or full KN
  - equil. curv.: $r \sim 0.6$, ($r_{3p} \sim 4.0$) for TH-KN

\[
n(\gamma) \propto \gamma^2 \exp \left( \frac{-1}{f(q, \dot{\gamma})} \left( \frac{\gamma}{\gamma_{eq}} \right)^{f(q, \dot{\gamma})} \right)
\]
Jet

\[ R \leq c \frac{\Delta t \delta}{(1+z)} \]

- \( \gamma - \gamma \) transparency
- \( B \)
- \( \gamma_{\text{max}} \)

BH

\[ R \leq c \frac{\Delta t}{(1+z)} \]

- \( M_{\text{BH}} \)
- disk/jet feeding

\( R \sim R_g \)
Jet

\[ R \leq c \frac{\Delta t \delta}{(1+z)} \]

\( \lambda_{\text{max}} \)

\( \rho_g \)

- \( \gamma \)-\( \gamma \) transparency
- \( B \)
- \( \gamma_{\text{max}} \)

BH

\[ R \leq c \frac{\Delta t}{(1+z)} \]

\( R \sim R_g \)

- \( M_{\text{BH}} \)
- disk/jet feeding
**$E_s$-$b_s$ X-ray trend and $\gamma$-ray predictions**

- The data span 13 years, both flaring and quiescent states.
- We are able to reproduce these long-term behaviours, by changing the value of only one parameter ($q$).
- Curvature values imply distribution far from the equilibrium ($b\sim[0.7-1.0]$).
- More data needed at GeV/TeV, curvature seems to be cooling-dominated.

![Diagram](image)

- $L_{\text{inj}} (E_s-b_s \text{ trend}) (\text{erg s}^{-1})$: $5 \times 10^{39}$
- $L_{\text{inj}} (E_s-L_s \text{ trend}) (\text{erg s}^{-1})$: $5 \times 10^{38}, 5 \times 10^{39}$
- $q$: $[3/2, 2]$
- $t_A (s)$: $1.2 \times 10^3$
- $t_{D0} = 1/D_{P0} (s)$: $[1.5 \times 10^4, 1.5 \times 10^5]$
- $T_{\text{inj}} (s)$: $10^4$
- $T_{\text{esc}} (R/c)$: 2.0
HBLs case

In the top right panel of Fig. 1, we show the trend of the trend obtained by fitting the numerical SED in the Optical band. For the SSC emission, the solid green circles represent the expected powerlaw trends predicted by Eq. 1b. The blue dashed line represents the trend predicted by Eq. 1a. For the HSPs scenario, the red line represents the synchrotron component, the green line represents the SSC component, and the dashed lines overlapping the synchrotron component represent the expected powerlaw trends predicted by Eq. 1b and Eq. 1c.

As in the top left panel, the bottom right panel shows the trend obtained by fitting the numerical SED in the Optical/UV band for the synchrotron component (red solid circles) and in the 433 MeV band for the SSC emission (solid green circles). The blue dashed line represents the trend predicted by Eq. 1a. For different values of $\gamma_{\text{min}}$, the relative deviation is of the order $10^{-18}$.

Typical HSPs SEDs suggest that EBL confounds the typical HSPs SEDs. For Fermi-LAT sources, the red line represents the synchrotron component, the green line represents the SSC component, and the dashed lines represent the expected powerlaw trends predicted by Eq. 1b and Eq. 1c.

As anticipated, the relative deviation is of the order $10^{-18}$, which is still acceptable to use. The HBLs case shows that the relative deviation is of the order $10^{-18}$, softer than those predicted by Eq. 1a. This bias allows the MC approach to give a more precise estimate of the SSC parameter space.
acceleration signature in the $E_S$-vs-$L_S$ trend

long-trend main drivers

Tramacere+2009

$$S_s(E_s) \propto n(\gamma_{3p})\gamma_{3p}^3 B^2 \delta^4$$

$$E_s \propto \gamma_{3p}^2 B \delta.$$ 

$$S_s \propto (E_s)^\alpha.$$ 

$n(\gamma_{3p}) \text{ const} \quad \alpha=1.5$

$n(\gamma_{3p}) \downarrow \Rightarrow \alpha<1.5$

acceleration+energy conservation

$\bullet \gamma_{3p} \uparrow \text{ and } n(\gamma_{3p}) \downarrow \Rightarrow \alpha<1.5$

$B \rightarrow \alpha=2.0$, incompatible as long-trend main driver

$\bullet \delta \rightarrow \alpha=4$
SEDs evolution

- S instant emission
- IC instant emission
- TH-KN transition
- S averaged emission
- IC averaged emission

$q=2$

$q=2.0, \delta B/B=0.1, \beta_A=0.5$

$q=1.5, \delta B/B=0.1, \beta_A=0.5$

$\tau_D$, $\tau_\gamma$, $\tau_c$, $\tau_\delta$, $t_{\text{acc}}$

$\gamma_{\text{max}}$

$\lambda_{\text{max}}$

$T_H$, $T_N$

$\Delta t$

$\lambda_{\text{inter}}$

$\gamma_c$

$\delta_B/B$
• Full bands curvature related to EED broadness, acceleration signature

• High energy band, dominated by cooling, moving towards the equilibrium

\[ r = \frac{c_e}{4D_{p0} t} \propto \frac{1}{D_{p0} t} \]
Moving Ep above 30 keV

Low cooling

- $B = 0.2/1.0$ G
- $R = 3 \times 10^{15}$ cm
- $L_{\text{inj}} = 5 \times 10^{39}$ erg/s
- $q = 2$
- $t_A = 1.2 \times 10^3$ s
- $t_D = 2.2 \times 10^4$ s

Strong cooling

- SEDs are rescaled in order that the **brightest** state matches the flux of $10^{-9}$ erg cm$^{-2}$ s$^{-1}$ [2-10] keV
- during the flares, the fluxes range in $\sim 1 \times 10^{-10}-10^{-9}$ erg cm$^{-2}$ s$^{-1}$
- 1 ks integration time
Effect of $B$ on SEDs

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- Setting its variation range to $[3, 10^3]$. The PL best fit of harder turbulence spectra, and the IC trend shows the transition component follows the expectation with a lower curvature for properties of these parameters are the same in both the TH to KN regime. The PL fit for energy lower values, compared to the "hard-sphere" case. The relations between the spectral parameters are very similar to those found in the previous case since diffusion, the curvature gets higher values and the peak value is dominated by the acceleration terms, while for values of $E_s \approx 2Gt$ the evolution of the spectral parameters. In Section 0.

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- In Section 0.

- In Section 0.
Rapid Variability

\[ R \leq c \frac{\Delta t}{\delta} \frac{1}{(1+z)} \]

\[ \Delta t = R/c \]

\[ \Delta t/\delta \]

\[ \Delta t(1+z) \]

\[ R \leq c \frac{\Delta t\delta}{(1+z)} \]
acceleration signature in the $E_s$-vs-$L_s$ trend

The relation between acceleration + energy conservation

\[ S_s(E_s) \propto n(\gamma_{3p})^{3} B^2 \delta^4 \]

\[ E_s \propto \gamma_{3p}^2 B \delta. \]

\[ S_s \propto (E_s)^\alpha. \]

\[ \alpha \approx 0.5 \]

\[ \alpha \approx 0.5 \]

\[ \alpha = 1.5 \]

\[ \gamma_{3p} \uparrow \text{ and } n(\gamma_{3p}) \downarrow \Rightarrow \alpha \leq 1.5 \]

acceleration + energy conservation

\[ B \rightarrow \alpha = 2.0, \quad \text{ incompatible as } \]

long-trend main driver

\[ \delta \rightarrow \alpha = 4 \]
The instantaneous SEDs at steps of 200 s: the solid lines represent the synchrotron and IC SEDs averaged over the 6s for comparison (top panel). All the other parameters are as reported in Table 1.

For the case of transition from TH to KN regime, the curvature is close to that of the synchrotron emission (purple) but with systematically lower values.

The synchrotron curvature quickly approaches the trend, as predicted by the Synchrotron model (blue trends). The IC curvature, since the transition from TH to KN regime in the IC curvature, since the curvature b3 ≈ 0.5 GeV (c2t0b2 (top panel), and b ≳ 0.5 (bottom panel). All the other parameters are as reported in Table 1.

5.2. E - approximation). The distribution of the diffusion coefficient as a function of the energy is the distribution of the equilibrium value of the diffusion coefficient for a detailed discussion). In the former approximation, the curvature b3 ≈ 0.5 GeV (c2t0b2 (top panel), and b ≳ 0.5 (bottom panel). All the other parameters are as reported in Table 1.

The synchrotron curvature quickly approaches the trend, as predicted by the Synchrotron model (blue trends). The IC curvature, since the transition from TH to KN regime in the IC curvature, since the curvature b3 ≈ 0.5 GeV (c2t0b2 (top panel), and b ≳ 0.5 (bottom panel). All the other parameters are as reported in Table 1.

The synchrotron curvature quickly approaches the trend, as predicted by the Synchrotron model (blue trends). The IC curvature, since the transition from TH to KN regime in the IC curvature, since the curvature b3 ≈ 0.5 GeV (c2t0b2 (top panel), and b ≳ 0.5 (bottom panel). All the other parameters are as reported in Table 1.
The IC spectra show an evident curvature that is not uniform across all energy values in the interval 0.50–1.90, the three curves correspond to different characteristic energies in the SED of a single zone SSC model. (Eq. (11)), computed for different values in the interval 0.50–1.90, the three curves correspond to different characteristic energies in the SED of a single zone SSC model. (Eq. (11)). In the upper panel, IC emission is dominated by Thomson scattering when the fraction of interactions are in the Klein-Nishina regime. We obtained spectra with a curvature that is not uniform across all energy values in the interval 0.50–1.90, the three curves correspond to different characteristic energies in the SED of a single zone SSC model. (Eq. (11)). In the upper panel, IC emission is dominated by Thomson scattering. The IC spectra show an evident curvature that is not uniform across all energy values in the interval 0.50–1.90, the three curves correspond to different characteristic energies in the SED of a single zone SSC model. (Eq. (11)). In the upper panel, IC emission is dominated by Thomson scattering.
D_{p}-driven trends \quad t_{D}=[1.5 \times 10^{4}-1.5 \times 10^{5}], \quad L_{\text{inj}}=\text{const.}
The evolution of energy distribution and curvature as a function of time and energy is shown for two different source sizes, $R = 10^{15}$ cm (top) and $R = 10^{13}$ cm (bottom). The graphs illustrate the effect of synchrotron cooling and the transition to Compton-dominated cooling. The vertical dot-dashed lines represent the equilibrium energy in the case of only synchrotron cooling. The graphs are labeled with parameter values for the numerical solutions of the diffusion equation, as detailed in Table 1. The horizontal and vertical axes represent the logarithm of the energy distribution and curvature, respectively, as a function of time.
effect of $\lambda_{\text{max}}$, $\lambda_{\text{coher}}$

$B=1.0$ G, $t_{D0}=1\times10^3$ s, $q=2.0$, $\lambda_{\text{max}}=10^9$ cm

$B=1.0$ G, $t_{D0}=1\times10^3$ s, $q=2.0$, $\lambda_{\text{max}}=10^{15}$ cm

$\text{synch. peak curvature}$

$b_{eq} \approx 1.0$

$b_{eq} \approx 0.7$